

Study of Anisotropic Compact Stars in Starobinsky Model

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Abstract

The aim of this paper is to study the formation of anisotropic compact stars in modified $f(R)$ theory of gravity, which is the generalization of the Einstein's gravity. To this end, we have used the solution of Krori and Barua to the anisotropic distribution of matter in $f(R)$ gravity. Further, we have matched the interior solution with the exterior solution to determine the constants of Krori and Barua solution. Finally the constant have been determined by using the data of compact stars like $4U1820 - 30$, $HerX - 1$, $SAX J1808 - 3658$. Using the evaluated form of the solutions, we have discussed the regularity of matter components at the center as well as on the boundary, energy conditions, anisotropy, stability analysis and mass-radius relation of the compact stars $4U1820 - 30$, $HerX - 1$, $SAX J1808 - 3658$.

Keywords: $f(R)$ Gravity Models; Compact stars

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1 Introduction

In the weak field regime, General Relativity (GR) has succeeded to counter the observation tests whereas strong field is yet to be explored. In fact the great success of GR has not stopped the alternatives being proposed and modifications begin to appear in very early days of this theory. Current investigations reveals the fact GR fails to explain the strong gravitational field effects which suggests that this theory may require modification. The presence of higher order term in Einstein-Hilbert (EH) action has motivated the researcher to modify this theory in strong field regime. In 1980, Starobinsky presented the idea of curvature driven inflationary scenario, where the action of GR is replaced by $f(R) = R + \lambda R^2$ [1]. In current years, numerous efforts have been made to go beyond the original Einstein theory, in order to discuss the accelerated expansion of the universe in more scientific ways [2].

Exploring the exotic compact objects in modified gravity, would be a scientific tool to handle this problem. The study of strong gravitational field of compact objects clearly explain the significant differences between GR and its modification. The modeling of massive star in $f(R)$ gravity have added some additional proprieties to stars [3, 4]. According to Psaltis [5] the strong gravitational fields could be considered as modified theories of gravity if we consider GR as the weak field limit of some more realistic effective gravitational theory. One can consider the stability of relativistic stars in $f(R)$ gravity as a test of the theory's viability; some $f(R)$ models do not allow the existence of stable star configurations and thus are considered unrealistic [6]. However, possible problems regarding the existence of these objects may be avoided due to the so-called Chameleon Mechanism [7]. The study of neutron stars in $f(R)$ gravity is currently an active field and people have worked on their existence as well as the stability [8]-[12].

During the last decades many researchers have derived the models of anisotropic compact stars. Egeland [13] discussed the modeling the mass-radius relation of the Neutron star and concluded that a cosmological constant would exist due to density of the vacuum. For this purpose, Egeland used the equation of hydrostatic equilibrium and fermion gas equation of state (EoS). Using spherical symmetry of compact stars, an exact solution of equation of was proposed by Mak and Harko [14], which predicts the properties of strange stars. Rahaman et al. [16] provided the extension of Krori-Barua [17] models using the Chaplygin gas EOS. Lobo [18] investigated the models of the compact objects with a barotropic EOS. He also

extended the Mazur-Mottola gravastar models by using the junction conditions between static spacetime and Schwarzschild vacuum solution. In the present study, we have investigated the formation of spherically symmetric anisotropic compact stars in $f(R)$ gravity that were initially suggested by Alcock et al. [19] and Haensel et al. [20]. The anisotropic compact stars models with linear equation of state and cvariable cosmological constant have been formulated by Hossein et al. [21].

We study the formation of anisotropic compact stars with more generalized $f(R)$ model i.e., $f(R) = R + \lambda R^2$ (where λ is constant) and conclude that $f(R)$ gravity can provide that existence the of anisotropic compact stars candidates X-ray bruster $4U1820 - 30$, $X - ray$ pulsar $HerX - 1$, Millisecond pulsar $SAX J1808 - 3658$. The objective of this paper is that if compact star solutions exist in $f(R)$, what are the constraints on $f(R)$ model and parameters of the theory? The spherically symmetric models of the compact stars proposed here are associated with $f(R)$ theory of gravity and we analyzed the stability of these models by using the anisotropic property of the model. This paper is organized as follow. In the coming section, we formulate the equations of motion for anisotropic source and static metric in $f(R)$ gravity. In Section 3, we discuss the implementation of the solution to a class of compact stars and present the physical behavior of the proposed models. In the last section, we summarize the findings of the paper.

2 Anisotropic Matter Configuration in $f(R)$ Gravity

The action of $f(R)$ theory of gravity in the presence of matter is given by [5]

$$\mathcal{I} = \int dx^4 \sqrt{-g} [f(R) + \mathcal{L}_{(matter)}], \quad (1)$$

where $8\pi G = 1$, R is the scalar curvature, $f(R)$ is an arbitrary function of R as well as its higher powers and $\mathcal{L}_{(matter)}$ denotes the Lagrangian density of matter part. Hence, we get the following form of field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}^{(curv)} + T_{\mu\nu}^{(matter)}, \quad (2)$$

where $T_{\mu\nu}^{(matter)}$ is the stress-energy tensor of the matter and $T_{\mu\nu}^{(curv)}$ is curvature term, given by

$$T_{\mu\nu}^{(curv)} = \frac{1}{F(R)} \left[\frac{1}{2} g_{\mu\nu} (f(R) - RF(R)) + F(R)^{;\alpha\beta} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\nu} g_{\alpha\beta}) \right], \quad (3)$$

where $F(R) = f'(R)$.

The general spherically symmetric metric is given by

$$ds^2 = -e^{\mu(r)} dt^2 + e^{\nu(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (4)$$

where $\nu = Ar^2$, $\mu = Br^2 + C$ [17], A , B and C are constants.

For the anisotropic fluid the energy-momentum tensor is defined by

$$T_{\alpha\beta}^m = (\rho + p_t) u_\alpha u_\beta - p_t g_{\alpha\beta} + (p_r - p_t) v_\alpha v_\beta, \quad (5)$$

where $u_\alpha = e^{\frac{\mu}{2}} \delta_\alpha^0$, $v_\alpha = e^{\frac{\nu}{2}} \delta_\alpha^i$, are four velocities, ρ is energy density, p_r and p_t are radial and transverse pressures, respectively. In this case set of field equations is

$$\begin{aligned} \rho &= -e^{-\nu} F'' + e^{-\nu} \left(\frac{\nu'}{2} - \frac{2}{r} \right) F' + \frac{e^{-\nu}}{r^2} \left(\frac{\mu'' r^2}{2} + \frac{\mu'^2 r^2}{4} - \frac{\mu' \nu' r^2}{4} + \mu' r \right) F \\ &\quad - \frac{1}{2} f, \end{aligned} \quad (6)$$

$$p_r = e^{-\nu} \left(\frac{\mu'}{2} + \frac{2}{r} \right) F' - \frac{e^{-\nu}}{r^2} \left(\frac{\mu'' r^2}{2} + \frac{\mu'^2 r^2}{4} - \frac{\mu' \nu' r^2}{4} - \nu' r \right) F + \frac{1}{2} f, \quad (7)$$

$$\begin{aligned} p_t &= -e^{-\nu} F'' + e^{-\nu} \left(\frac{\mu'}{2} - \frac{\nu'}{2} + \frac{1}{r} \right) F' - \frac{e^{-\nu}}{r^2} \left(\frac{\mu' r}{2} - \frac{\nu' r}{2} - e^\nu + 1 \right) F \\ &\quad + \frac{1}{2} f. \end{aligned} \quad (8)$$

The Starobinsky model is [1]

$$f(R) = R + \lambda R^2, \quad (9)$$

where λ is an arbitrary constant. The most important thing in existence of compact stars is the requirement of static configuration i.e., the EoS satisfies the condition $\rho - 3p > 0$. Therefor, in fixing λ , one needs to analyze this situation and avoid the existence of singularities. In this settings we find that

the viable values of λ lies in the range $0 < \lambda < 6$. One can choose suitable value of λ according to this condition. Herein we set $\lambda = 2km^2$.

For this model the equation (6)-(8) become

$$\begin{aligned} \rho = & \frac{e^{-2\nu}}{8r^4} \{ r^4 \lambda \mu'^4 - 2r^3 \lambda \mu'^3 (-4 + r\nu') + r^2 \lambda \mu'^2 (16 + 8r\nu' - 11r^2 \nu'^2 + 4r^2 \mu'' \\ & + 8r^2 \nu'') + 4r^2 \lambda \mu' (-16r\nu'^2 + 3r^2 \nu'^3 + \nu' (-4 + 9r^2 \mu'' - 7r^2 \nu'') + 2r(-2\mu'' \\ & + 6\nu'' - 2r\mu''' + r\nu''')) + 4(-2e^\nu r^2 + 2e^{2\nu} r^2 - 20\lambda + 24e^\nu \lambda - 4e^{2\nu} \lambda \\ & + 12r^3 \lambda \nu'^3 - 3r^4 \lambda \mu''^2 - r^2 \lambda \nu'^2 (12 + 11r^2 \mu'') + 8r^2 \lambda \nu'' + 8r^4 \lambda \mu'' \mu'' - 16 \\ & \times r^3 \lambda \mu''' + 2r\nu'(e^\nu r^2 - 8\lambda + 16r^2 \lambda \mu'' - 14r^2 \lambda \nu'' + 6r^3 \lambda \mu''') + 8r^3 \lambda \nu''' \\ & - 4r^4 \lambda \mu^{(iv)}) \}, \end{aligned} \quad (10)$$

$$\begin{aligned} p_r = & \frac{e^{-2\nu}}{8r^4} \{ -r^4 \lambda \mu'^4 - 2r^4 \lambda \mu'^3 \nu' + r^3 \lambda \mu'^2 (-24\nu' + 3r\nu'^2 + 4r(\mu'' - \nu'')) \\ & - 4(-2e^\nu r^2 + 2e^{2\nu} r^2 + 28\lambda - 24e^\nu \lambda - 4e^{2\nu} \lambda - 12r^2 \lambda \nu'^2 - 16r^2 \lambda \mu'' \\ & + 8r^3 \lambda \nu' \mu'' + r^4 \lambda \mu''^2 + 16r^2 \lambda \nu'' - 8r^3 \lambda \mu''' + 8r\mu'(e^\nu r^2 - 8\lambda + 3r^2 \lambda \nu'^2 \\ & + 6r^2 \lambda \mu'' - r\lambda\nu'(8 + r^2 \mu'') - 4r^2 \lambda \nu'' + r^3 \lambda \mu''') \}, \end{aligned} \quad (11)$$

$$\begin{aligned} p_t = & \frac{e^{-\nu}}{8r^4} \{ r^4 \lambda \mu'^4 + 2r^3 \lambda \mu'^3 (2 - 3r\nu') + r^2 \mu'^2 (-32r\lambda\nu' + 17r^2 \lambda \nu'^2 + 2(e^\nu r^2 \\ & - 8\lambda + 6r^2 \lambda \mu'' - 6r^2 \lambda \nu'')) - 2r\mu'(-38r^2 \lambda \nu'^2 + 6r^3 \lambda \nu'^3 + r\nu'(e^\nu r^2 - 24\lambda \\ & + 28r^2 \lambda \mu'' - 14r^2 \lambda \nu'')) - 2(e^\nu r^2 - 4\lambda + 12e^\nu \lambda + 10r^2 \lambda \mu'' - 14r^2 \lambda \nu'' \\ & + 6r^3 \lambda \mu''' - 2r^3 \lambda \nu''') - 4(12r^3 \lambda \nu'^3 - 11r^4 \lambda \mu'' \nu'^2 - 5r^4 \lambda \mu''^2 + \mu''(-e^\nu r^4 \\ & + 8r^2 \lambda + 8r^4 \lambda \nu'')) + r\nu(e^\nu r^2 - 28\lambda + 12e^\nu \lambda + 28r^2 \lambda \mu'' - 28r^2 \lambda \nu'' + 12r^3 \\ & \times \lambda \mu''') + 4\lambda(-7 + 6e^\nu + e^{2\nu} - 3r^3 \mu''' + 2r^3 \nu''' - r^4 \mu^{(iv)}) \}. \end{aligned} \quad (12)$$

We have five unknown functions ρ, p_r, p_t, μ, ν , and three Eqs.(10)-(12). We have to chose any two functions, keeping in mind the regularity conditions of the compact stars, we chose μ and ν as after Eq.(4). As metric functions are exponential i.e., $e^{\mu(r)}, e^{\nu(r)}$, for $\nu = Ar^2$, $\mu = Br^2 + C$, metric functions remain exponential as well as regular even at center of the star. Moreover, this choice satisfies the boundary conditions in the center of the star [12]

$$\rho(0) = \rho_c, \quad \nu(0) = 0, \quad \frac{d\mu}{dr}(0) = 0.$$

From the metric potential function, we get following form of matter com-

ponents

$$\begin{aligned}\rho &= \frac{1}{r^4} e^{-2Ar^2} \{ e^{2Ar^2} (r^2 - 2\lambda) + 2(-5 - 3B^2r^4 + 6B^3r^6 + B^4r^8 + 12A^3r^6 \\ &\times (2 + Br^2) - A^2r^4(40 + 68Br^2 + 11B^2r^4) + A(-4r^2 + 48Br^4 \\ &+ 26B^2r^6 - 2B^3r^8))\lambda + e^{Ar^2}(-r^2 + 2Ar^4 + 12\lambda)\},\end{aligned}\quad (13)$$

$$\begin{aligned}p_r &= \frac{1}{r^4} e^{-2Ar^2} \{-e^{2Ar^2}(r^2 - 2\lambda) + 2(-7 + 11B^2r^4 + 2B^3r^6 - B^4r^8 \\ &+ 3A^2r^4(2 + Br^2)^2 - 2Ar^2(4 + 16Br^2 + 9B^2r^4 + B^3r^6))\lambda + e^{Ar^2}(r^2 \\ &+ 2Br^4 + 12\lambda)\},\end{aligned}\quad (14)$$

$$\begin{aligned}p_t &= \frac{1}{r^4} e^{-2Ar^2} \{-2(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda + 16B^3r^6\lambda + 2B^4r^8\lambda - 24A^3r^6(2 \\ &+ Br^2)\lambda + 2A^2r^4(28 + 74Br^2 + 17B^2r^4)\lambda + B^2r^4(e^{Ar^2}r^2 + 22\lambda) + 2Br^2 \\ &\times (-6\lambda + e^{Ar^2}(r^2 + 6\lambda)) - Ar^2(4(-7 + 19Br^2 + 25B^2r^4 + 3B^3r^6)\lambda \\ &+ e^{Ar^2}(r^2 + Br^4 + 12\lambda))\}.\end{aligned}\quad (15)$$

Here, we consider the following form of linear equation of state (EOS)

$$p_r = w_r \rho, \quad p_t = w_t \rho \quad (16)$$

The above equations lead to the following relations

$$\begin{aligned}\omega_r &= \{-e^{2Ar^2}(r^2 - 2\lambda) + 2(-7 + 11B^2r^4 + 2B^3r^6 - B^4r^8 + 3A^2r^4(2 + Br^2)^2 \\ &- 2Ar^2(4 + 16Br^2 + 9B^2r^4 + B^3r^6))\lambda + e^{Ar^2}(r^2 + 2Br^4 + 12\lambda)\} / \{e^{2Ar^2} \\ &\times (r^2 - 2\lambda) + 2(-5 - 3B^2r^4 + 6B^3r^6 + B^4r^8 + 12A^3r^6(2 + Br^2) - A^2r^4 \\ &\times (40 + 68Br^2 + 11B^2r^4) + A(-4r^2 + 48Br^4 + 26B^2r^6 - 2B^3r^8))\lambda \\ &+ e^{Ar^2}(-r^2 + 2Ar^4 + 12\lambda)\},\end{aligned}\quad (17)$$

$$\begin{aligned}\omega_t &= \{-2(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda + 16B^3r^6\lambda + 2B^4r^8\lambda - 24A^3r^6(2 + Br^2)\lambda \\ &+ 2A^2r^4(28 + 74Br^2 + 17B^2r^4)\lambda + B^2r^4(e^{Ar^2}r^2 + 22\lambda) + 2Br^2(-6\lambda \\ &+ e^{Ar^2}(r^2 + 6\lambda)) - Ar^2(4(-7 + 19Br^2 + 25B^2r^4 + 3B^3r^6)\lambda + e^{Ar^2}(r^2 \\ &+ Br^4 + 12\lambda))\} / \{e^{2Ar^2}(r^2 - 2\lambda) + 2(-5 - 3B^2r^4 + 6B^3r^6 + B^4r^8 \\ &+ 12A^3r^6(2 + Br^2) - A^2r^4(40 + 68Br^2 + 11B^2r^4) + A(-4r^2 + 48Br^4 \\ &+ 26B^2r^6 - 2B^3r^8))\lambda + e^{Ar^2}(-r^2 + 2Ar^4 + 12\lambda)\}.\end{aligned}\quad (18)$$

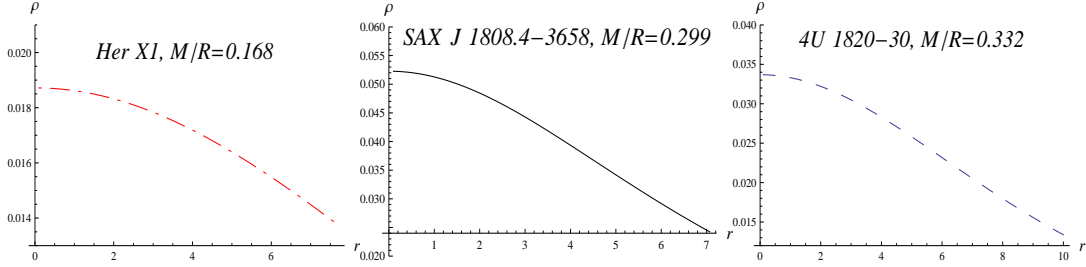


Figure 1: Variation of energy density ρ versus radial coordinate $r(km)$. Herein, we set $\lambda = 2km^2$.

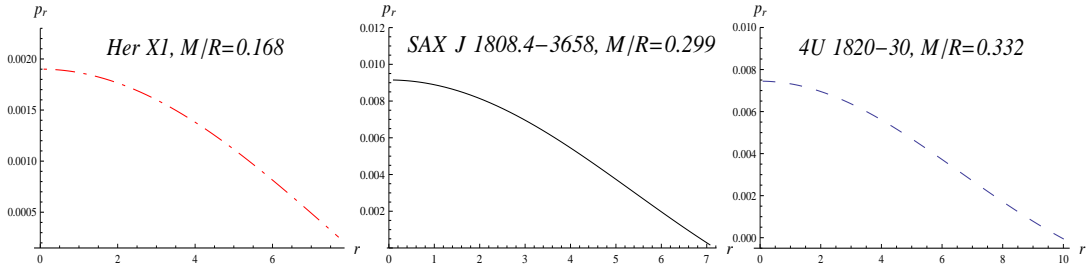


Figure 2: Variation of radial pressure p_r versus radial coordinate $r(km)$.

3 Physical Analysis

In this section, we discuss the following physical properties of the solutions.

3.1 Anisotropic Constraints

In the first place, we present the evolution of energy density ρ , radial pressure p_r and tangential pressure p_t as shown in Figures **1-3** for different strange stars (see Table 1).

Taking derivatives of equations (13) and (14) with respect to radial coor-

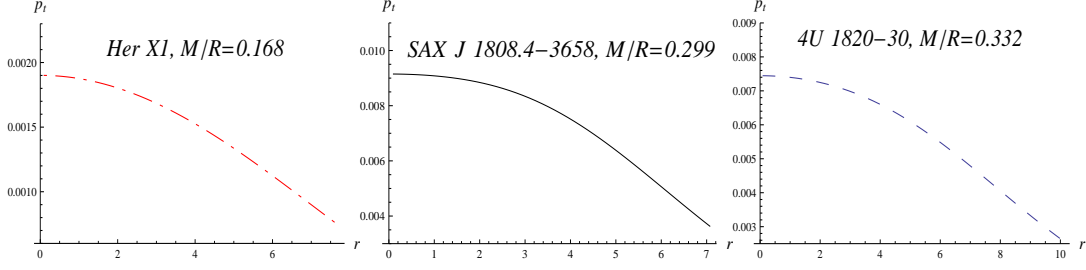


Figure 3: Variation of transverse pressure p_t versus radial coordinate $r(km)$

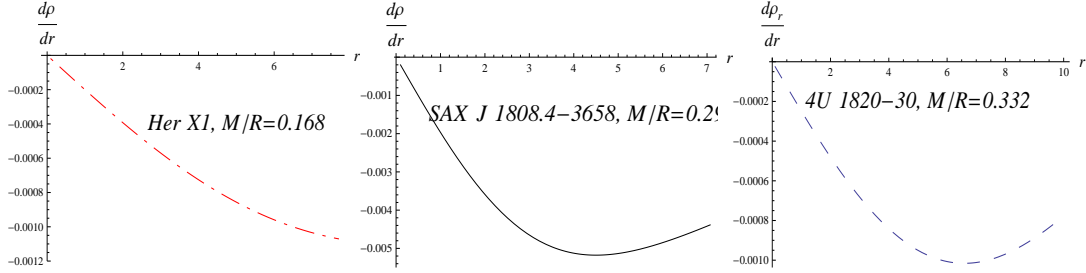


Figure 4: Behavior of $\frac{d\rho}{dr}$ versus radial coordinate $r(km)$.

dinate, we have

$$\begin{aligned} \frac{d\rho}{dr} = & \frac{1}{r^5} 2e^{-2Ar^2} \{ -e^{2Ar^2} (r^2 - 4\lambda) - 4(-5 - 3B^3r^6 - B^4r^8 + 12A^4r^8(2 \\ & + Br^2) - A^3r^6(52 + 80Br^2 + 11B^2r^4) + A^2r^4(-4 + 82Br^2 + 37B^2r^4 \\ & - 2B^3r^6) + Ar^2(-7 - 16B^2r^4 + 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(Ar^4 - 2A^2r^6 \\ & - 24\lambda + r^2(1 - 12A\lambda)) \}, \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{dp_r}{dr} = & \frac{1}{r^5} 2e^{-2Ar^2} \{ e^{2Ar^2} (r^2 - 4\lambda) - 4(-7 - B^3r^6 + B^4r^8 + 3A^3r^6(2 + Br^2)^2 \\ & - A^2r^4(8 + 38Br^2 + 21B^2r^4 + 2B^3r^6) + Ar^2(-11 + 20B^2r^4 + 4B^3r^6 \\ & - B^4r^8))\lambda - e^{Ar^2}(Ar^4 + 2ABr^6 + 24\lambda + r^2(1 + 12A\lambda)) \}. \end{aligned} \quad (20)$$

The evolution of $\frac{d\rho}{dr}$ and $\frac{dp_r}{dr}$ is shown in Figures 4 and 5. It can be seen that $\frac{d\rho}{dr} < 0$ and $\frac{dp_r}{dr} < 0$.

We also examine the behavior of derivatives of ρ and p_r at center $r = 0$

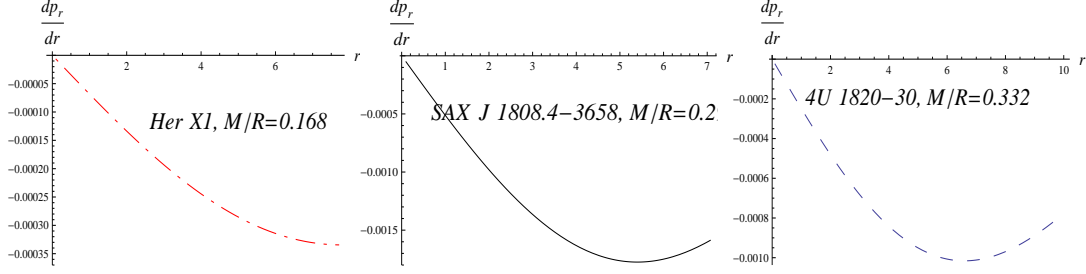


Figure 5: Behavior of $\frac{dp_r}{dr}$ versus radial coordinate $r(km)$.

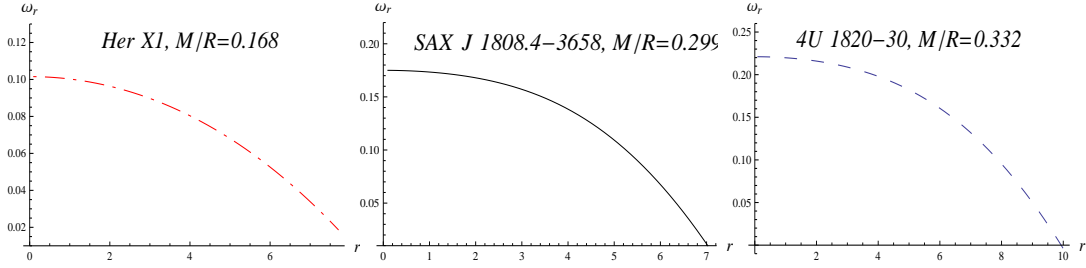


Figure 6: Variation of EoS parameter ω_r versus radial coordinate $r(km)$.

of compact star and it is found that

$$\begin{aligned} \frac{d\rho}{dr} &= 0, & \frac{dp_r}{dr} &= 0, \\ \frac{d^2\rho}{dr^2} &< 0, & \frac{d^2p_r}{dr^2} &< 0. \end{aligned} \quad (21)$$

Equation (21) shows the maximality of central ρ and p_r . Hence ρ and p_r attain maximum values at $r = 0$ and functional values decreases with the increase in r as shown in Figures **1-3**. We present the evolution of EoS parameters ω_r and ω_t in Figures **6** and **7** for different strange stars. We call these parameters as effective since these involve the contribution from the additional terms in $f(R)$ gravity. Here, it is clear that, like normal matter distribution, the bound on the effective EOS in this case is given by $0 < \omega_i(r) < 1$, ($i = r, t$).

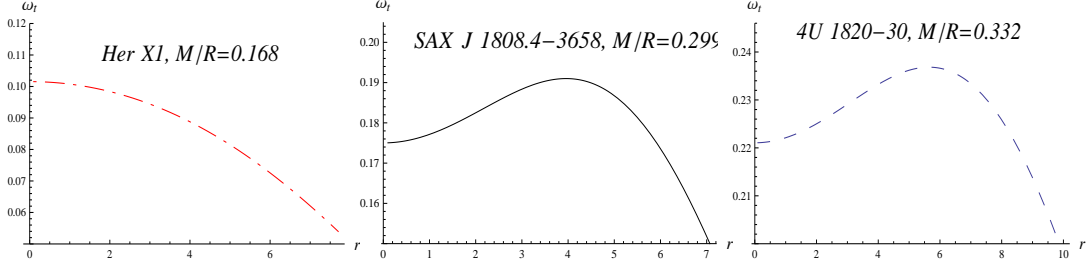


Figure 7: Variation of EoS parameter ω_t versus radial coordinate $r(km)$.

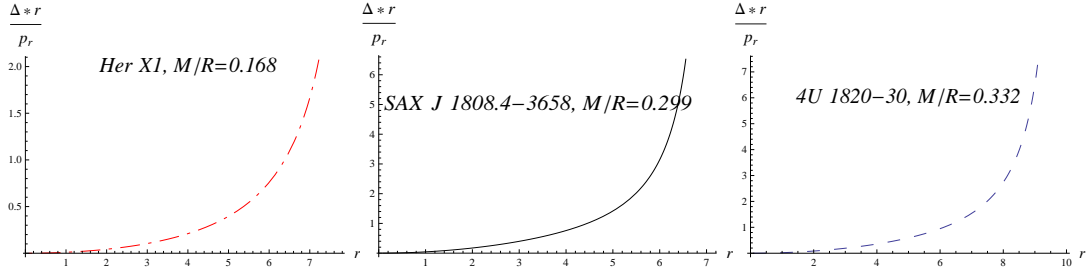


Figure 8: Variation of anisotropy measurement .

The anisotropy measurement $\Delta = \frac{2}{r}(p_t - p_r)$ for this model is given by

$$\begin{aligned} \Delta = & \frac{1}{r^5} 2e^{-2Ar^2} \{ e^{2Ar^2} (r^2 - 4\lambda) - 4(-7 + 3Br^2 - 3B^3r^6 - B^4r^8 + 6A^3r^6(2 \\ & + Br^2) - A^2r^4(8 + 31Br^2 + 7B^2r^4) + Ar^2(-11 + 3Br^2 + 16B^2r^4 + 2B^3 \\ & \times r^6))\lambda - e^{Ar^2}(Ar^4 + (A - B)Br^6 + 24\lambda + r^2(1 + 12A\lambda - 12B\lambda)) \}. \end{aligned} \quad (22)$$

The measure of anisotropy is directed outward when $p_t > p_r$ which implies $\Delta > 0$ whereas it is directed inward if $p_t < p_r$ resulting in $\Delta < 0$. In this discussion we consider the fractional pressure anisotropy given by $\Delta r/p_r$. The evolution of fractional pressure anisotropy is shown in Figure 8. It is obvious that $\Delta r/p_r$ remains positive at the stellar interior of strange star candidates. Hence for this case repulsive force exists which allows the construction of more massive configuration.

It is interesting to see that anisotropy vanishes at the center $r = 0$ and the corresponding pressures take the form $p_t(0) = p_r(0) = p_0 = 34A^2\lambda + 2B(1 + 11B\lambda) - A(1 + 64B\lambda)$.

3.2 Matching Conditions

In [25], Cooney et al. studied the formation of compact objects like Neutron Star in $f(R)$ gravity theories with perturbation constraints. The Schwartzchild-Sitter metric is considered as an exterior solution which is matched with the interior spherical symmetry using conditions analogous to that in GR. According to these authors [22]-[24] Schwarzschild solution is the most suitable solution as exterior geometry of the star. Using this approach, a lot of work has been done [8, 9, 10, 11, 12, 22]-[28] by taking Schwarzschild or Vaidya metric to address the problems related to gravitational collapse and neutron stars in $f(R)$ gravity.

The vacuum exterior spherically symmetric metric given by

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad (23)$$

The continuity of the metric functions g_{tt} , g_{rr} and $\frac{\partial g_{tt}}{\partial r}$ yield,

$$g_{tt}^- = g_{tt}^+, \quad g_{rr}^- = g_{rr}^+, \quad \frac{\partial g_{tt}^-}{\partial r} = \frac{\partial g_{tt}^+}{\partial r}, \quad (24)$$

where $-$ and $+$, are quantities for the internal and external portion of star. Hence we get

$$A = -\frac{1}{R^2} \ln \left(1 - \frac{2M}{R}\right), \quad (25)$$

$$B = \frac{M}{R^3} \left(1 - \frac{2M}{R}\right)^{-1}, \quad (26)$$

$$C = \ln \left(1 - \frac{2M}{R}\right) - \frac{M}{R} \left(1 - \frac{2M}{R}\right)^{-1}. \quad (27)$$

Li et al. [29] studied X-ray pulsar SAX J1808.4-3658 to compare its mass-radius relation with theoretical mass-radius relation of strange star and for neutron star candidates and shown the consistency of strange star model with SAX J1808.4-3658. They suggested that SAX J1808.4-3658 is a likely strange star candidate and calculated masses and radii of strange star as $1.44M_\odot$, $1.32M_\odot$ and $7.07km$, $6.53km$, respectively. Zhang et al. [30] presented the mass measurement for the neutron star in 4U 1820-30 and reported mass of the order $\simeq 2.2M_\odot$. In [31], mass and radius of neutron star in 4U 1820-30

are determined with 1σ error as $M = 1.58 \pm 0.06M_\odot$ and a radius of $R = 9.11 \pm 0.4km$. However, upper bound limit in this measurement is consistent with that in [30]. In fact there is a certain uncertainty in measurement of mass and radius of a compact stars. Abubekero et al. [29] estimated the mass of Her X-1 using more recent and physically justified techniques and found two different values of masses $m_x = 0.85 \pm 0.15M_\odot$ and $m_x = 1.8M_\odot$ through the radial-velocity curves. This uncertainty may be due to the tense X-ray heating in Her X-1. For M and R [29]-[34] of the compact stars, the constants A and B are given in the table 1.

Table 1: Values of constants for given Masses and Radii of Stars

Strange Quark Star	M	$R(km)$	$\frac{M}{R}$	$A(km^{-2})$	$B(km^{-2})$
<i>HerX</i> – 1	$0.88M_\odot$	7.7	0.168	0.0069062764281	0.0042673646183
<i>SAX J1808.4</i> – 3658	$1.435M_\odot$	7.07	0.299	0.018231569740	0.014880115692
<i>4U1820</i> – 30	$2.25M_\odot$	10.0	0.332	0.010906441192	0.0098809523811

3.3 Energy Conditions

The validity of these energy conditions is necessary for a physically reasonable energy-momentum tensor. The energy conditions for anisotropic fluid are defined by the following relations

$$NEC : \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad (28)$$

$$WEC : \quad \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad (29)$$

$$SEC : \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad \rho + p_r + 2p_t \geq 0, \quad (30)$$

$$DEC : \quad \rho > |p_r|, \quad \rho > |p_t|. \quad (31)$$

In Figure 9 energy conditions are fulfilled for our model.

3.4 TOV Equation

The generalized Tolman-Oppenheimer-Volkoff (TOV) equation gets the form

$$\frac{dp_r}{dr} + \frac{\nu'(\rho + p_r)}{2} + \frac{2(p_r - p_t)}{r} = 0 \quad (32)$$

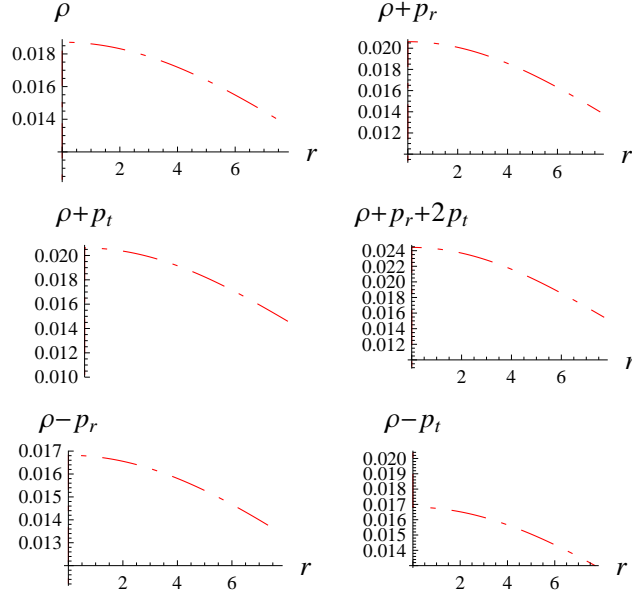


Figure 9: Evolution of energy constraints for compact star Her X-1.

Following [21], above equation can be written as

$$F_g + F_h + F_a = 0, \\ F_g = -Br(\rho + p_r), \quad F_h = -\frac{dp_r}{dr}, \quad F_a = \frac{2(p_t - p_r)}{r} \quad (33)$$

Using the effective ρ , p_r and p_t (10)-(12), for strange star Her X-1, we have plotted these values in figure 10.

3.5 Stability Analysis

People [35]-[37] have discussed the appearance of cracking in spherical compact objects by using different approaches. Herrera [35] introduced the concept of cracking to identify potentially unstable anisotropic matter configuration. It was considered to explain the behavior of fluid distribution, once the equilibrium configuration has been perturbed and total non-vanishing radial forces of different signs appear within the system. Now, by considering the sound speeds one can assess the potentially stable and unstable regions established through the difference of sound propagation within the matter

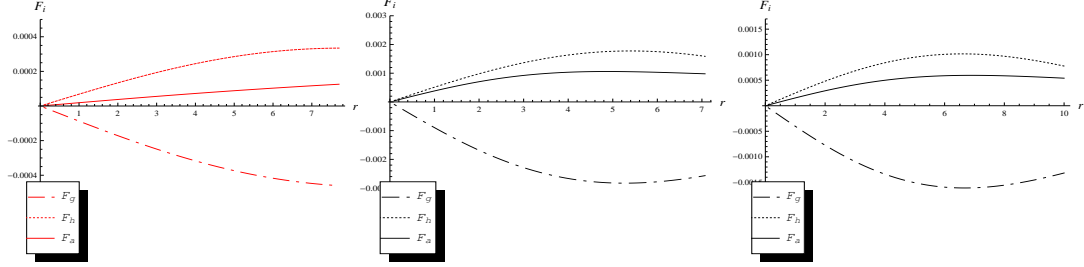


Figure 10: Variation of gravitating, hydrostatic and pressure anisotropic forces for compact star candidates.

configuration. The region for which radial sound of sound v_{sr}^2 is greater than the transverse speed of sound v_{st}^2 is potentially stable.

To analyze the stability of our model we calculate the radial and transverse speeds as

$$\begin{aligned}
 v_{sr}^2 &= \{-e^{2Ar^2}(r^2 - 4\lambda) + 4(-7 - B^3r^6 + B^4r^8 + 3A^3r^6(2 + Br^2)^2 - A^2r^4 \\
 &\quad \times (8 + 38Br^2 + 21B^2r^4 + 2B^3r^6) + Ar^2(-11 + 20B^2r^4 + 4B^3r^6 \\
 &\quad - B^4r^8))\lambda + e^{Ar^2}(Ar^4 + 2ABr^6 + 24\lambda + r^2(1 + 12A\lambda))\}/\{e^{2Ar^2}(r^2 \\
 &\quad - 4\lambda) + 4(-5 - 3B^3r^6 - B^4r^8 + 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2 \\
 &\quad + 11B^2r^4) + A^2r^4(-4 + 82Br^2 + 37B^2r^4 - 2B^3r^6) + Ar^2(-7 - 16B^2r^4 \\
 &\quad + 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(-Ar^4 + 2A^2r^6 + 24\lambda + r^2(-1 + 12A\lambda))\}, \quad (34) \\
 v_{st}^2 &= \{B^2e^{Ar^2}r^6 + 4(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda - 12B(-1 + e^{Ar^2})r^2\lambda + 16B^3r^6\lambda \\
 &\quad + 4B^4r^8\lambda + 48A^4r^8(2 + Br^2)\lambda - 4A^3r^6(40 + 86Br^2 + 17B^2r^4)\lambda + A^2r^4 \\
 &\quad \times (4(-14 + 75Br^2 + 67B^2r^4 + 6B^3r^6)\lambda + e^{Ar^2}(r^2 + Br^4 + 12\lambda)) - Ar^2 \\
 &\quad \times (-8(-7 + 3e^{Ar^2})\lambda + 56B^3r^6\lambda + 4B^4r^8\lambda + B^2r^4(e^{Ar^2}r^2 + 144\lambda) + 3Br^2 \\
 &\quad \times (-8\lambda + e^{Ar^2}(r^2 + 4\lambda)))\}/\{-e^{2Ar^2}(r^2 - 4\lambda) - 4(-5 - 3B^3r^6 - B^4r^8 \\
 &\quad + 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2 + 11B^2r^4) + A^2r^4(-4 + 82Br^2 \\
 &\quad + 37B^2r^4 - 2B^3r^6) + Ar^2(-7 - 16B^2r^4 + 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(Ar^4 \\
 &\quad - 2A^2r^6 - 24\lambda + r^2(1 - 12A\lambda))\}. \quad (35)
 \end{aligned}$$

In Figures 11 and 12 it is shown that v_{sr}^2 and v_{st}^2 satisfy the inequalities $0 \leq v_{sr}^2 \leq 1$ and $0 \leq v_{st}^2 \leq 1$ within the anisotropic matter configuration.

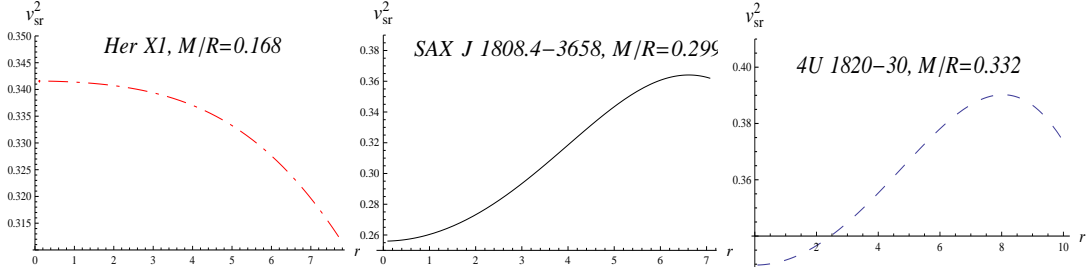


Figure 11: Variation of v_{sr}^2 for compact star candidates.

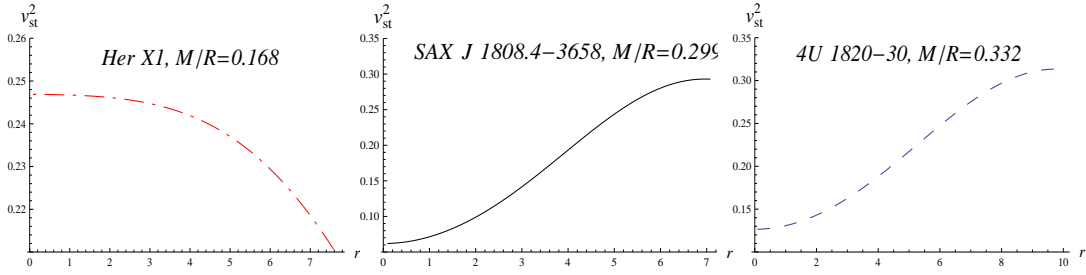


Figure 12: Variation of v_{st}^2 for compact star candidates.

The difference of v_{sr}^2 and v_{st}^2 can be obtained as

$$\begin{aligned}
v_{st}^2 - v_{sr}^2 = & \{e^{2Ar^2}(r^2 - 8\lambda) - 4(-14 + 3Br^2 + 3B^3r^6 + 2B^4r^8 + 12A^4r^8(2 \\
& + Br^2) - 2A^3r^6(14 + 37Br^2 + 7B^2r^4) + A^2r^4(-22 + 37Br^2 + 46 \\
& \times B^2r^4 + 4B^3r^6) - Ar^2(25 - 6Br^2 + 16B^2r^4 + 10B^3r^6 + 2B^4r^8)) \\
& \times \lambda - e^{Ar^2}((A^2 - AB + B^2)r^6 + A(A - B)Br^8 + 48\lambda + Ar^4(1 \\
& + 12A\lambda - 12B\lambda) + r^2(1 + 36A\lambda - 12B\lambda))\} / \{e^{2Ar^2}(r^2 - 4\lambda) + 4 \\
& \times (-5 - 3B^3r^6 - B^4r^8 + 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2 \\
& + 11B^2r^4) + A^2r^4(-4 + 82Br^2 + 37B^2r^4 - 2B^3r^6) + Ar^2(-7 \\
& - 16B^2r^4 + 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(-Ar^4 + 2A^2r^6 + 24\lambda \\
& + r^2(-1 + 12A\lambda))\}. \tag{36}
\end{aligned}$$

The $v_{st}^2 - v_{sr}^2$ of different strange stars is shown in Figure 13. Thus, our proposed model is stable.

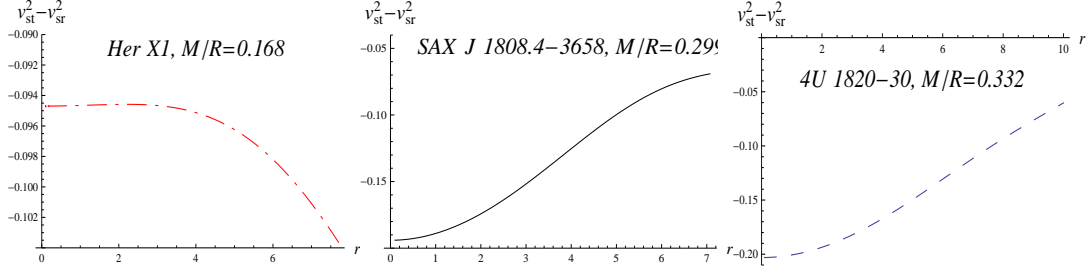


Figure 13: Variation of $v_{st}^2 - v_{sr}^2$ for compact star candidates.

3.6 Surface Redshift

The Mass-radius relation is

$$\begin{aligned}
 u &= \frac{M(R)}{R} = \frac{\pi e^{-2AR^2}}{32A^2(AR^2)^{3/2}} \{ -4\sqrt{AR^2}(15R^2B^4\lambda + 192A^5R^4(2 + R^2B)\lambda + 2AR^2 \\
 &\times B^3(21 + 10R^2B)\lambda - 16A^4R^2(22 + 53R^2B + 11R^4B^2)\lambda + A^2R^2B^2(99 \\
 &+ 56R^2B + 16R^4B^2)\lambda - 4A^3(-49R^4B^2\lambda + 8R^6B^3\lambda + 16(5 - 6e^{AR^2} \\
 &+ e^{2AR^2})\lambda + R^2(-8e^{AR^2} + 8e^{2AR^2} - 33B\lambda))) - 1536A^4R^2e^{2AR^2} \text{Erf}(\sqrt{AR^2}) \\
 &\times \sqrt{\pi}\lambda + 3R^2(224A^4 + 44A^3B + 33A^2B^2 + 14AB^3 + 5B^4)e^{2AR^2} \sqrt{2\pi}\lambda \\
 &\times \text{Erf}(\sqrt{2AR^2}) \}. \tag{37}
 \end{aligned}$$

The surface redshift (Z_s) is

$$\begin{aligned}
 1 + Z_s &= (1 - 2u)^{-1/2} = \{ 1 - \frac{\pi e^{-2AR^2}}{16A^2(AR^2)^{3/2}} \{ -4\sqrt{AR^2}(15R^2B^4\lambda + 192A^5R^4 \\
 &\times (2 + R^2B)\lambda + 2AR^2B^3(21 + 10R^2B)\lambda - 16A^4R^2(22 + 53R^2B + 11R^4 \\
 &\times B^2)\lambda + A^2R^2B^2(99 + 56R^2B + 16R^4B^2)\lambda - 4A^3(-49R^4B^2\lambda + 8R^6B^3 \\
 &\times \lambda + 16(5 - 6e^{AR^2}e^{2AR^2})\lambda + R^2(-8e^{AR^2} + 8e^{2AR^2} - 33B\lambda))) - 1536A^4 \\
 &\times R^2e^{2AR^2} \text{Erf}(\sqrt{AR^2})\sqrt{\pi}\lambda + 3R^2(224A^4 + 44A^3B + 33A^2B^2 + 14AB^3 \\
 &+ 5B^4)e^{2AR^2} \sqrt{2\pi}\lambda \text{Erf}(\sqrt{2AR^2}) \} \}^{-1/2}. \tag{38}
 \end{aligned}$$

Figure 13 shows the plot of redshift of compact star Her X-1 of radius 7 km and the maximum redshift turns out to be $Z_s = 0.845$.

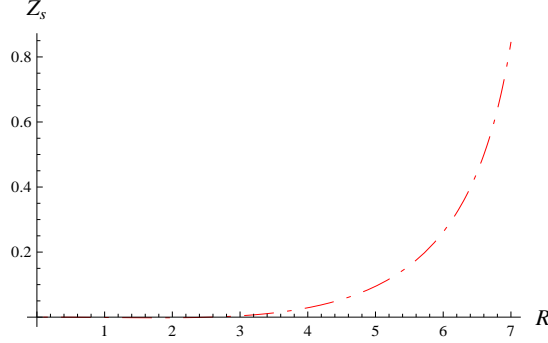


Figure 14: Surface redshift of Her X-1.

4 Conclusion

The modified $f(R)$ theory of gravity providing the theoretical explanation of accelerated expansion of universe, has attracted the much attention of modern cosmologist. This theory has attained a particular interest since the $f(R)$ modifications to general theory of relativity appeared in a very natural way in the low-energy effective actions of the quantum theory of gravity and the quantization of underlying fields in curved spacetime. This theory is also conformally related to GR with some exotic scalar field [38].

This paper deals with the study of anisotropic compact stars whose interior source is static. To complete the study, we have considered that there may exists such compact stars that have anisotropy in their interiors in the framework of $f(R)$ gravity. The interior geometry of the compact stars has been handled by metric assumption proposed by Krori and Barua [17]. Then we perform the matching of the interior metric with exterior Schwarzschild metric to determine the constants of interior metric in terms of masses and radii of the compact stars. The application of the masses and radii of the compact stars yield the values of constants that determine the nature of the stars. For these values of the constants, we found that the energy conditions hold for the given class of compact strange stars. By the physical interpretation of the results, we conclude that the EOS parameters are given by $0 < \omega_i(r) < 1$, ($i = r, t$). This indicates that fact that compact stars are composed of ordinary matter and effect of $f(R)$ gravity term. The matter components remains finite and positive every where inside the stars and attain the maximum value at the center. Thus our considered compact stars

models are singularity free.

It is interesting to note that anisotropic force will be directed outward when $P_t > P_r$ this implies that $\Delta > 0$. We have found that $\Delta > 0$ for the different strange stars as shown in Figure 8. Hence, in this case repulsive force exists which allows the construction of more massive stellar configuration in $f(R)$ gravity. The subliminal velocity of sound is less than 1, i.e, $0 < v_{sr}^2$, $v_{st}^2 < 1$ and $v_{sr}^2 > v_{st}^2$. The variation of $v_{st}^2 - v_{sr}^2$ for different strange stars is shown in Figure 13, which satisfies the inequality $|v_{st}^2 - v_{sr}^2| \leq 1$. Thus, in the presence of $f(R)$ term the constructed compact stars models are stable. The range of surface redshift Z_s for the class of the particular star is $0 < Z_s \leq 0.845$. The analysis of the compact stars in GR in the absence of cosmological constant implies that redshift is $Z_s \leq 2$. Therefore, we conclude that in the present situation redshift has been reduced to a certain value.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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